The Emperor's New Constants

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The unreasonable tailor

In 1960, Eugene Wigner wrote an essay that has haunted physics ever since. He called it “The Unreasonable Effectiveness of Mathematics in the Natural Sciences.” His puzzle: why should abstract mathematical structures, developed with no reference to physical reality, turn out to describe that reality with such precision? Riemann develops the geometry of curved spaces in 1854, doing pure mathematics. Einstein picks it up sixty years later and it fits the structure of spacetime exactly. Lie develops group theory to study symmetries of differential equations. Physicists find it describes the symmetries of fundamental particles.

Wigner called it a miracle. He left it unexplained. Which, if you think about it, should have been more alarming than it was.

But consider the tailor. A tailor who always produces clothes that fit perfectly is either very skilled or measuring the customer first. Which is physics doing?

The answer, once you look carefully, is unsettling. Physics has been measuring the customer, running experiments, extracting numbers, finding patterns, and then visiting the mathematics cupboard to find clothes that fit. The fit is always good enough. The clothes are often beautiful. But the process is fitting, not deriving. The mathematics is borrowed. The correspondence between the mathematical object and the physical phenomenon is a human choice, made after the fact.

Wigner’s miracle is not a miracle. It is the inevitable result of a century of careful fitting. This essay is about what happens when you stop fitting and start deriving. About what E = mc² and 1/137 look like when the clothes come off.

The cupboard

Modern physics has a cupboard. Riemann gave a lecture in 1854 on the geometry of curved spaces, doing pure mathematics with no physical application in mind. Sixty years later, Einstein needed a mathematical language for general relativity. He visited the cupboard. Riemannian geometry fit. One correspondence choice: the curvature of spacetime is gravity. After that, all of Riemannian geometry became physically applicable.

Sophus Lie developed group theory to study symmetries of differential equations. Physicists built the Standard Model around SU(3)×SU(2)×U(1), a choice of Lie group that fits the observed particle spectrum. It was not derived. It was selected. From the cupboard. Again.

Each time: mathematics developed independently, physics finding it later, a human correspondence choice declaring the fit. That gap, the set of human choices connecting mathematics to the world, is the correspondence layer. Every physical theory has one. Its thickness is measurable: count the free choices. The Standard Model has twenty-six.

The gap

Newton says F = ma. Apply a force of one Newton to a mass of one kilogram and you get an acceleration of one metre per second squared. Verified countless times. But what is a Newton? It is defined as one kilogram metre per second squared. What is a kilogram? Originally, a specific platinum-iridium cylinder kept in a vault near Paris. What is a metre? Originally one ten-millionth of the distance from the equator to the North Pole.

Good questions. Follow them to the end. The equation F = ma relates three quantities whose definitions are human artefacts. A physicist on a different planet with different conventions would write the same law with different numbers. The mathematics is the same. The law is the same. But the correspondence between the symbols and physical quantities runs through a thick layer of human choices.

This is not a flaw in Newton's law. It is the nature of physical modelling. The equation is true within the measurement framework you chose. But the measurement framework is not reality. It is a human interface to reality.

Now consider what it would look like if the gap were not there. Not thinner. Absent. A circle is the set of all points equidistant from a centre. A circular orbit is the path of an object maintaining constant distance from a gravitational centre. These are not two things in correspondence. They are the same thing. The circle does not model the orbit. The orbit is a circle because both are the same underlying relation, instantiated in different contexts. No gap.

Mathematics is the study of pure relations. Reality is a structure of relations. When the relations match exactly, there is no gap. The mathematics does not model the reality. They are the same relational structure, encountered from two directions.

The famous equation

$E = mc^2$. Written on coffee mugs, painted on walls. The most famous equation in physics. Ask anyone what it means and they will say: energy equals mass times the speed of light squared. Matter can be converted to energy. The atomic bomb. Nuclear power. The sun. All true. All correct. All beautifully confirmed by experiment.

Now let us look at the clothes.

The speed of light, c = 299,792,458 metres per second. That number depends entirely on the metre and the second. The metre is currently defined as the distance light travels in 1/299,792,458 of a second, which is circular. The number 299,792,458 is a human artefact: the speed of light in a unit system invented by French revolutionaries based on the size of the Earth. A physicist using Planck units writes E = m. Just that. The c^2 vanishes. Not because it was wrong, but because it was always a conversion factor between human measurement conventions.

Strip the equation of its units entirely. Here is what remains: matter is light that has stopped propagating. When light freezes into a standing wave, when the propagating mode becomes self-referential and forms a closed loop, it pays a cost. That cost is its mass. The exchange rate between propagating energy and standing energy is set by the propagation bandwidth: c. Squared, because the exchange involves both the outward and the return leg of the self-referential loop.

The famous equation is true. It is also attire. And has been for over a century. The body beneath it is:

Frozen coherence carries the energy of propagating coherence, converted at the rate set by self-reference.

No units. No free constants. Just the relation.

The greatest mystery

1/137.036. Richard Feynman called it “one of the greatest mysteries of physics.” He said all good theoretical physicists put this number on their wall and worry about it. It is the fine structure constant α, the coupling constant of electromagnetism. It appears in the energy levels of hydrogen, in the magnetic moment of the electron, in the probability of any electromagnetic interaction. It is dimensionless: a pure number with no units, independent of any human measurement convention.

And nobody had ever derived it from first principles. Not in a century of trying. A century. Think about that.

Eddington famously claimed to derive it as 1/136, then revised to 1/137 when the measurement changed. That is not a derivation. That is fitting with extra steps. Wyler produced a formula in 1971 involving volumes of symmetric spaces in higher dimensions, accurate to six decimal places. But the formula was constructed by searching through combinations of known mathematical constants until something fit. Dozens of similar attempts exist. None eliminated the free choices in the correspondence layer.

So 1/137 sat in the foundation of atomic physics as a measured input, a brute fact, an emperor with no clothes. Real. Precise. Governing every atomic interaction. And nobody knew why.

Or rather: nobody knew why. Past tense.

The clothes that were never there

Sometimes physics builds an entire ontology around the fit, declaring that because the model works, the things in the model must exist. This is where the Emperor's clothes become genuinely dangerous. Not just attire worn over reality, but attire mistaken for reality itself.

Quarks. The quark model fits the observed spectrum of hadrons with extraordinary elegance. Six quarks, three colours, fractional charges. The model worked. It predicted new particles before they were found. There is one problem. No quark has ever been observed. Not once. Not ever. Every experiment designed to isolate a free quark has failed. Physics responded by declaring confinement a fundamental feature and proceeding as though quarks were real objects that happened to be stuck. But the model fit because it found the right symmetry, not because it found the right objects. The quarks are the symmetry structure, misread as particles. In Pentagon Physics, there are no quark particles. There is a proton: six eigenvalues of the 600-cell adjacency operator in three-phase balance, confined by the Galois boundary, with mass derived from the axiom.

Gravity. Newton's gravitational constant G: measured, inserted, fundamental. Every calculation involving gravity takes G as an input. Nobody has ever explained where G comes from. Not Newton. Not Einstein. Not string theory. Not loop quantum gravity. In Pentagon Physics, G is derived. The gravitational coupling $\alpha_G = \alpha^{18} \times 12/7$. It is the exhaust of confinement: the leakage through the 102 unoccupied vertices of the 600-cell that the proton does not use. G is not a fundamental constant. It is a consequence: the shadow of confinement, seen from the outside. Which is a rather different thing.

Nuclear masses. In 1935, von Weizsäcker wrote down a formula for nuclear binding energy with five fitted terms. The formula works. It reproduces binding energies across the periodic table. But it never answered the deepest question: why does the binding energy curve peak at iron? Why do both fusion and fission release energy, and why do both directions point to the same element? The Pentagon Physics answer: iron is the nearest element to the fixed point of the axiom. One equation with no free parameters replaces five fitted coefficients. And guess what, it explains the shape, not just the numbers.

In every case: the clothes were well-made. The body was more interesting than the clothes suggested.

What things are both mathematics and reality

Some things are simultaneously mathematical and real in a way that most things are not. The circle. The integer. Symmetry. Prime numbers. Self-reference. In each case, the mathematical object and the physical phenomenon are not connected by a correspondence choice. They are the same thing.

What do these have in common? They are all purely relational. A circle is a relation: equal distance from a point. An integer is a relation: position in a sequence. A symmetry is a relation: invariance under a transformation. None of them require substance. None require units. They are structures of pure relation, and they are real in the deepest sense: not real because they exist somewhere, but real because they cannot be otherwise.

Mathematics is the study of pure relations. Reality, at its most fundamental, is also relational structure, not made of stuff but made of how things relate. When the mathematical relation and the physical relation are the same relation, there is no gap. And it suggests a question: what would physics look like if it operated entirely at this level? If instead of fitting mathematical relations to physical phenomena, it derived both from the same underlying relational structure?

It would look like an axiom.

The axiom

There is exactly one self-referential equation with no free parameters:

$$\sigma = 1/(1 + \sigma)$$

This says: σ is equal to one divided by one plus itself. It is the only equation of this form where the coefficient is 1. Any other coefficient introduces a free parameter and breaks the uniqueness. This equation does not describe self-reference. It is self-reference. The simplest statement a mathematical object can make about its own structure, with no external input.

It has one positive solution: $\sigma = 1/\phi$, where $\phi = (1+\sqrt{5})/2$. The Galois group of this equation is the automorphism $\sqrt{5} \to -\sqrt{5}$. Évariste Galois built his theory precisely to study polynomial equations of this kind. He did not know he was studying the axiom of a physical theory. He was twenty years old when he wrote it down, the night before a duel he did not survive. One correspondence choice: the universe is self-referential. After that, mathematics takes over completely.

The new clothes

From the axiom, by necessity: the discriminant is 5. This forces the algebraic field $\mathbb{Q}(\sqrt{5})$. The geometry of five-fold symmetry in four dimensions is the 600-cell, a regular polytope with 120 vertices. Its symmetry group is the binary icosahedral group 2I. Its adjacency matrix has nine distinct real eigenvalues, all in $\mathbb{Q}(\sqrt{5})$.

The Galois automorphism acts on these eigenvalues. Two survive the filter: $\lambda = 12$ and $\lambda = 6\phi$. These are gravity and electromagnetism, the only two long-range forces. Seven are frozen: these correspond to the Standard Model parameters. The number of long-range forces is not a model choice. It is the count of sign-alternating conjugate pairs in the spectrum of the 600-cell. It was always going to be two.

And now: 1/137. No search. No fitting. No correspondence choice after the axiom. The assembled formula:

$$\alpha^{-1} = 360/\varphi^2 - 2/\varphi^3 + 3^{-5}/\varphi^5 + 7^{-7}/\varphi^7 = 137.035999207$$

The experimental value is 137.035999206. The deviation is 0.05 standard deviations. The greatest mystery in physics is a theorem of self-reference. It was always going to be 137.036. Not because it was chosen. Because the axiom has discriminant 5 and the pentagon has interior angles of 108° and there was never any other value it could take.

Really. Never any other value.

The naked emperor

The constants were never mysterious. The equation was never fundamental.

$E = mc^2$ is true. 1/137 is real. The Standard Model's 26 parameters are measured with extraordinary precision. None of this changes. The attire was well-made. The tailors were brilliant: Riemann, Lie, Hilbert, Maxwell, Einstein, Dirac. Each found the right clothes for the physics they were studying. The cupboard is a remarkable achievement of human mathematical intelligence.

But the Emperor, the relational structure underneath, was always there. It did not need to be dressed. It needed to be looked at directly.

A single self-referential equation. One positive solution. One algebraic field. One automorphism. A geometry. A spectrum. A filter. Two forces. Seven frozen parameters. And from those parameters: 137.036. And from 137.036: $E = mc^2$.

Mathematics is not unreasonably effective at describing reality. Mathematics and reality are the same subject, relational structure, encountered from two directions. The physicist walks toward it through experiment. The mathematician walks toward it through proof. They keep finding each other because they are walking toward the same thing.

The correspondence layer does not vanish. It thins to one. One claim: the universe is self-referential. After that, there is no model. There is only the structure, unfolding from the inside.

The Emperor was never wearing clothes. We were.